For Supervisor's use only

3

90636

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2005

90636 Integrate functions and solve problems by integration, differential equations or numerical methods

Credits: Six
9.30 am Wednesday 16 November 2005

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables booklet L3-CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

Show the results of any integration needed to solve the problems.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Integrate functions and solve problems by integration, differential equations or numerical methods.	Find integrals and use integration to solve problems.	Use a variety of integration techniques to solve problem(s).
	Overall Level of Performance	

You are advised to spend 50 minutes answering the questions in this booklet.

Assessor's use only

Show **ALL** working.

QUESTION ONE

Find the integrals.
You do not need to simplify your answers.

Do not forget the arbitrary constant.

(a) $\int 3e^{2x-4}\,dx$

(b) $\int -\operatorname{cosec}^2 3x\,dx$

(c) $\int \frac{7x+4}{x}\,dx$

Assessor's use only

QUESTION TWO

Scientists find a large cave in the Abel Tasman National Park.

They wish to calculate the cross-sectional area of the cave at a point where the cave is 80 m wide.

They measure the height of the roof of the cave above the floor at 10 metre intervals across this width.

This table shows these heights.

Distance (m)	0	10	20	30	40	50	60	70	80
Height (m)	0	11.4	13.2	10.8	15.1	19.8	20.1	12.9	0

Use Simpson's Rule to estimate the cross-sectional area of the cave in m^2.

QUESTION THREE

Assessor's use only

Solve the differential equation: $\frac{dy}{dx} = y(2x + 1)$, given that $y = 120$ when $x = 1.4$.

QUESTION FOUR

Assessor's use only

Find the integrals:

(a) $\int 12x\sqrt{2x-4}\,\mathrm{d}x$. A suitable substitution may be helpful.

(b) $\int (8\cos 3x \cos x)\,\mathrm{d}x$.

QUESTION FIVE

Assessor's use only

Radioactive by-products are taken away and stored in lead containers until the radioactivity decays to a safe level.

The rate of change of radiation is proportional to the amount of radiation (N) present after t days.

After 20 days of storage, the radiation count is 120 rads.

After a further 12 days, the radiation count is 85 rads.

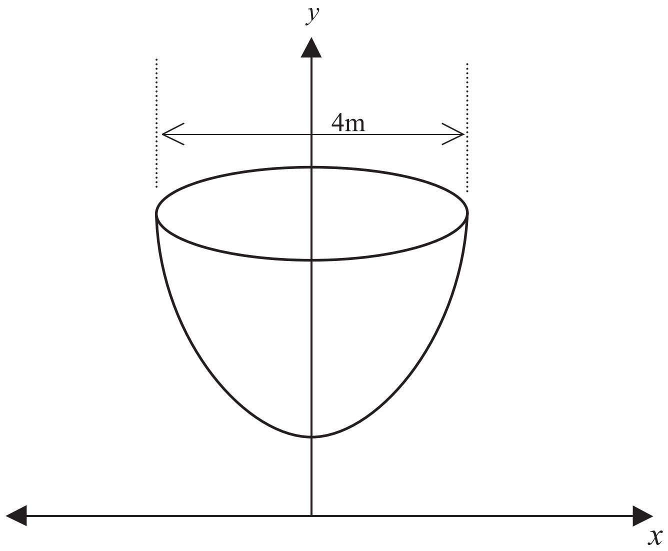
The by-products are safe to move if the radiation count is 30 or less.

Write a differential equation and solve it to calculate the length of time in days from when the by-products are stored in the lead containers until they are safe to move.

QUESTION SIX

Assessor's use only

The Olympic Flame is in a large bowl on a stand.



The shape of the bowl can be found by rotating part of the curve $y = x^2 + 3$ through 360° around the y-axis.

The bowl is 4 m wide at the top.

Calculate the volume in m^3 enclosed within the bowl.

You must show the results of any integration needed to solve this problem.

QUESTION SEVEN

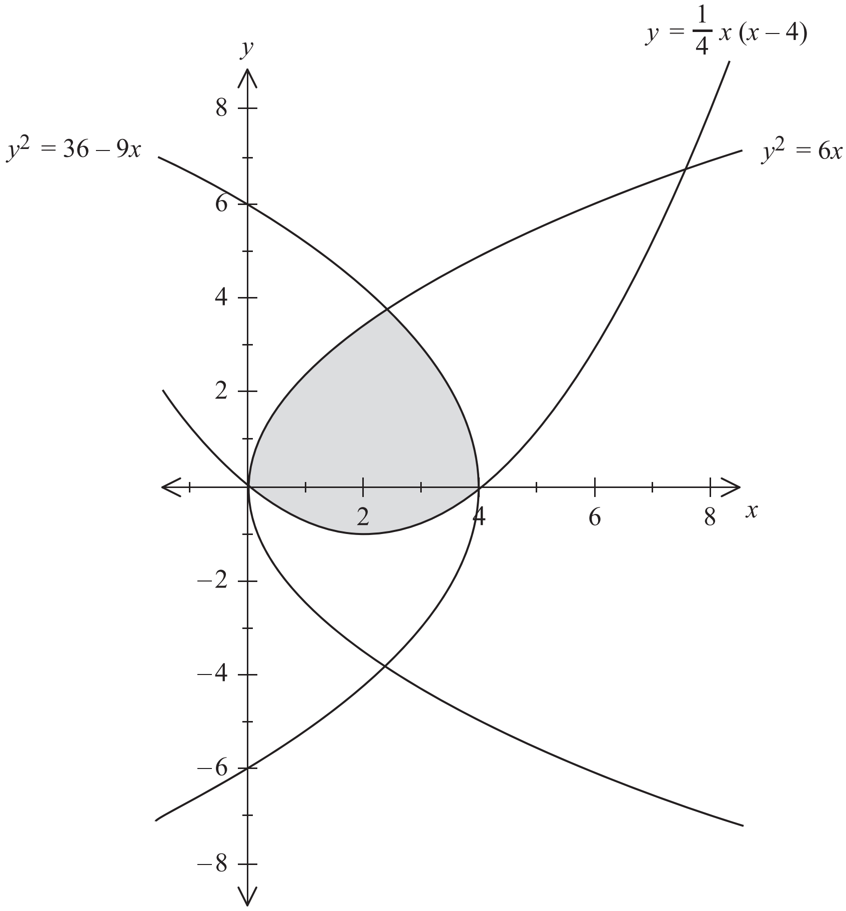
Assessor's use only

The shaded region in the diagram below is bounded by the three parabolas:

$y^2 = 36 - 9x$

$y = \frac{1}{4}x(x - 4)$

$y^2 = 6x$



Find the area of the shaded region.

Assessor's use only

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number